Short Tricks - JEE-Main

By

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1. Quadratic Equation

Lecture - 1

Q.1 If roots of the equation $(a-b)x^2 + (c-a)x + (b-c) = 0$ are equal, then a, b, c are in -

(A) A.P. (B) H.P. (C) G.P. (D) None of these

Sol. **[A]**

Proper Method	**Short Trick**
Given $(a-b)x^2 + (c-a)x + (b-c) = 0$ Roots are equal $\therefore D = 0$ $(c-a)^2 - 4(a-b)(b-c) = 0$ $\Rightarrow c^2 + a^2 - 2ac - 4(ab - ac - b^2 + bc) = 0$ $\Rightarrow c^2 + a^2 + 4b^2 + 2ac - 4ab - 4bc = 0$ $\Rightarrow (c + a - 2b)^2 = 0$ $\Rightarrow c + a - 2b = 0$ $\Rightarrow 2b = a + c$ $\therefore$ a, b, c are in A.P.	$a - b + c - a + b - c = 0$ $\therefore a - b = b - c$ $\Rightarrow 2b = a + c$ So a, b, c, are in A.P.

Q.2 If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then a, b, c are in -

(A) HP (B) GP (C) AP (D) None of these

(A) HP (B) GP (C) AP (D) None of these

Sol. **[A]**

Proper Method

Given :

$a\,(b-c)\,x^2 + b\,(c-a)\,x + c\,(a-b) = 0$

Roots are equal

$\therefore D = 0$

$\Rightarrow b^2(c-a)^2 - 4a(b-c)\,c(a-b) = 0$

$\Rightarrow b^2(c^2 + a^2 - 2ac) - 4ac(ba - b^2 - ca + bc) = 0$

$\Rightarrow b^2c^2 + b^2a^2 - b^2\,2ac - 4a^2\,bc + 4acb^2 + 4a^2c^2 - 4abc^2 = 0$

$\Rightarrow b^2c^2 + b^2a^2 + 4a^2c^2 + 2acb^2 - 4a^2bc - 4abc^2 = 0$

$\Rightarrow (bc + ba - 2ac)^2 = 0$

$\Rightarrow bc + ba - 2ac = 0$

$\Rightarrow b = \dfrac{2ac}{a+c}$

$\therefore$ a, b, c are in H.P.

Short Trick

$a(b-c) + b(c-a) + c(a-b) = 0$

$\therefore a(b-c) = c(a-b)$

$\Rightarrow b = \dfrac{2ac}{a+c}$

So, a, b, c are in H.P.

Q.3 If a, b, c are distinct positive real numbers such that $b(a + c) = 2ac$, then the roots of $ax^2 + 2bx + c = 0$ are:

(A) Real and equal (B) Real and distinct (C) Imaginary (D) None of these

Sol. [C]

Proper Method	**Short Trick**
$D = 4b^2 - 4ac = 4\left\{\dfrac{4a^2c^2}{(a+c)^2} - ac\right\}$ $= \dfrac{16ac}{(a+c)^2}\{4ac - (a+c)^2\}$ $= -\dfrac{16ac}{(a+c)^2}(a-c)^2 < 0$ [$\because$ a, c > 0] $\Rightarrow$ roots are imaginary	$b = \dfrac{2ac}{a+c} \Rightarrow a, b, c$ are in H.P. let $a = 2, b = 3, c = 6$ now equation $2x^2 + 6x + 6 = 0$ $D = 36 - 48 < 0$ (imaginary roots)

Q.4 If a, b, c, $\in$ R and 1 is a root of the equation $ax^2 + bx + c = 0$, then the equation $4ax^2 + 3bx + 2c = 0$, $c \neq 0$ has roots which are :

(A) Real and equal (B) Real and distinct (C) Imaginary (D) Rational

Sol. [B]

Proper Method	**Short Trick**
1 is a root of $ax^2 + bx + c = 0 \Rightarrow a + b + c = 0$ D of $4ax^2 + 3bx + 2c = 0$ is $= 9b^2 - 32ac = 9(a + c)^2 - 32ac$ $= c^2\left\{9\left(\frac{a}{c}\right)^2 - 14\left(\frac{a}{c}\right) + 9\right\}$ $= c^2\left\{\left(3\left(\frac{a}{c}\right) - \left(\frac{7}{3}\right)\right)^2 + 9 - \frac{49}{9}\right\} > 0$ $c^2\left\{3\left(\frac{a}{c} - \frac{7}{3}\right)^2 + \frac{31}{9}\right\} > 0$ $\Rightarrow$ roots are real and distinct.	Let the roots are 1 and 2 then equation $ax^2 + bx + c = 0$ becomes $x^2 - 3x + 2 = 0$ $(a = 1, b = -3, c = 2)$ Now equation $4ax^2 + 3bx + 2c = 0$ $\Rightarrow 4x^2 - 9x + 4 = 0$ $D = 81 - 64 > 0$ (real and distinct)

Q.5 If α and β are the roots of the equation $x^2 - p(x + 1) - q = 0$, then the value of

$$\frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + q} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + q}$$ is :

(A) 2 (B) 1 (C) 0 (D) None

Sol. **[B]**

Proper Method	**Short Trick**
Equation is $x^2 - px - (p + q) = 0$ $\alpha + \beta = p,\ \alpha\beta = -(p + q)$ Now $(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1$ $= -(p + q) + p + 1 = 1 - q$ The given expression $= \dfrac{(\alpha+1)^2}{(\alpha+1)^2+(q-1)} + \dfrac{(\beta+1)^2}{(\beta+1)^2+(q-1)}$ $= \dfrac{[2(\alpha+1)^2(\beta+1)^2+(q-1)\{(\alpha+1)^2+(\beta+1)^2\}]}{[(\alpha+1)^2(\beta+1)^2+(q-1)\{(\alpha+1)^2+(\beta+1)^2\}+(q-1)^2]}$ $= \dfrac{2(1-q)^2+(q-1)[(\alpha+1)^2+(\beta+1)^2]}{2(1-q)^2+(q-1)[(\alpha+1)^2+(\beta+1)^2]} = 1$	Let $\alpha = 1$ and $\beta = 2$, then equation is $x^2 - 3x + 2 = 0$ so $p = 3$ and $-p - q = 2$ $\Rightarrow q = -5$ $\dfrac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+q} + \dfrac{\beta^2+2\beta+1}{\beta^2+2\beta+q}$ $= \dfrac{4}{-2} + \dfrac{9}{3} = 1$

Q.6 If tan A and tan B are the roots of $x^2 + ax + b = 0$, then the value of expression $\sin^2(A + B) + a \sin(A + B) \cos(A + B) + b \cos^2(A + B)$ is equal to -

(A) $\frac{a}{b}$ (B) $\frac{b}{a}$ (C) a (D) b

Sol. [D]

Proper Method

$\tan A + \tan B = -a$ and $\tan A . \tan B = b$

$$\Rightarrow \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B} = \frac{-a}{1-b} = \frac{a}{b-1}$$

$$\Rightarrow \sin(A+B) = \frac{a}{\sqrt{a^2 + (b-1)^2}}$$

and $\cos(A+B) = \frac{b-1}{\sqrt{a^2 + (b-1)^2}}$

$\sin^2(A+B) + a \sin(A+B)\cos(A+B) + b\cos^2(A+B)$

$$= \frac{a^2}{a^2 + (b-1)^2} + \frac{a^2(b-1)}{a^2 + (b-1)^2} + \frac{b(b-1)^2}{a^2 + (b-1)^2}$$

$$= \frac{a^2 + a^2 b - a^2 + b^3 - 2b^2 + b}{a^2 + (b-1)^2}$$

$$= \frac{b(a^2 + b^2 - 2b + 1)}{(a^2 + b^2 - 2b + 1)}$$

$= b$

Short Trick

Let $A = 30°$ and $B = 60°$, then roots are $\frac{1}{\sqrt{3}}, \sqrt{3}$

and equation is $x^2 - \frac{4}{\sqrt{3}}x + 1 = 0$

so, $a = -\frac{4}{\sqrt{3}}$ and $b = 1$

now $\sin^2(A+B) + a\sin(A+B)\cos(A+B) +$
$b\cos^2(A+B) = 1 + 0 + 0 = 1 = b$

Q.7 If α, β are the roots of the equation $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$, then $a\,S_{n+1} + c\,S_{n-1} =$

(A) bS_n (B) b^2S_n (C) $2bS_n$ (D) $-bS_n$

Sol. [D]

Proper Method	**Short Trick**
Here α, β are roots $\therefore \quad a\alpha^2 + b\alpha + c = 0 \quad ...(1)$ $a\beta^2 + b\beta + c = 0 \quad ...(2)$ Now let us consider (Keeping results (1), (2) in mind) $a S_{n+1} + b S_n + c S_{n-1}$ $= a[\alpha^{n+1} + \beta^{n+1}] + b[\alpha^n + \beta^n] + c[\alpha^{n-1} + \beta^{n-1}]$ $= [a\alpha^{n+1} + b\alpha^n + c\alpha^{n-1}] + [a\beta^{n+1} + b\beta^n + c\beta^{n-1}]$ $= \alpha^{n-1}[a\alpha^2 + b\alpha + c] + \beta^{n-1}[a\beta^2 + b\beta + c]$ $= 0 + 0 = 0$ Hence $aS_{n+1} + cS_{n-1} = -bS_n$.	Let $\alpha = 1, \beta = 2$, then the equation $x^2 - 3x + 2 = 0$ so, $a = 1, b = -3, c = 2$ and let $n = 2 \Rightarrow S_2 = \alpha^2 + \beta^2 = 5$ now $aS_{n+1} + cS_{n-1}$ $= S_3 + 2S_1 = 9 + 6 = 15$ by option (D) $-bS_n = (+3)(5) = 15$ is correct